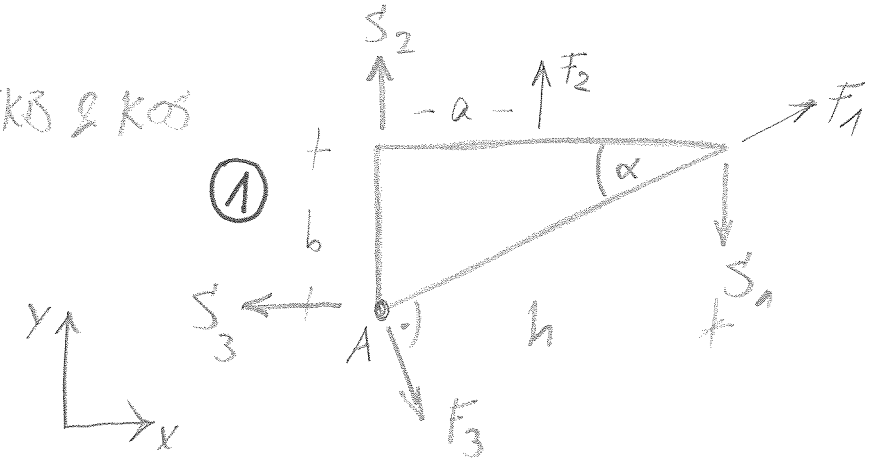
MUSTERLÖSUNG & PUNKTE // Techn. Mech. // 06.02.'14

// Berger

1. FKB & KOS



①

$$\tan\alpha = \frac{b}{h}$$

$$\rightarrow h = \frac{b}{\tan\alpha}$$

② $\uparrow$: $-S_1 + S_2 + F_1 \sin\alpha + F_2 - F_3 \cos\alpha = 0$ (1)

② $\rightarrow$: $-S_3 + F_1 \cos\alpha + F_3 \sin\alpha = 0$ (2)

② $\curvearrowleft A$: $-F_2 a + S_1 \dfrac{b}{\tan\alpha} = 0$ (3)

3 Glch. / 3 Unbek. ✓

- aus (3): $S_1 = F_2 \dfrac{a}{b} \tan\alpha$
- aus (2): $S_3 = F_1 \cos\alpha + F_3 \sin\alpha$
- alles in (1): $S_2 = S_1 - F_1 \sin\alpha - F_2 + F_3 \cos\alpha$

②

$$= F_2 \frac{a}{b} \tan\alpha - F_1 \sin\alpha - F_2 + F_3 \cos\alpha$$

$$= F_2 \left[\frac{a}{b} \tan\alpha - 1\right] - F_1 \sin\alpha + F_3 \cos\alpha$$

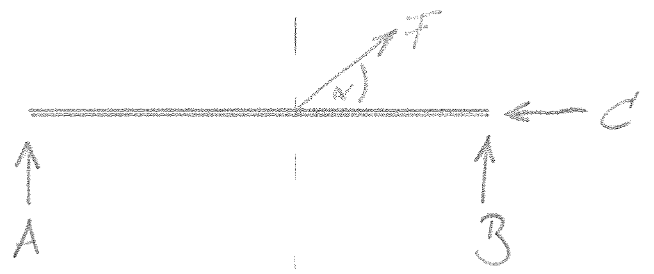
$\alpha = 30°$: $\sin\alpha = \frac{1}{2}$ $\cos\alpha = \frac{1}{2}\sqrt{3}$ $\tan\alpha = \sqrt{3}/3$

$\alpha = 60°$: $\sin\alpha = \frac{1}{2}\sqrt{3}$ $\cos\alpha = \frac{1}{2}$ $\tan\alpha = \sqrt{3}$ ①

$\rightarrow$ Einsetzen liefert S_1, S_2 und S_3

2.

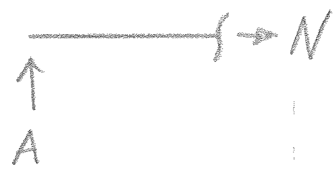
i)



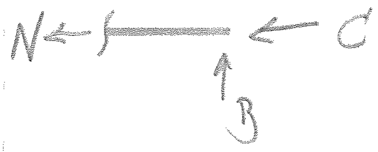
GG-Bed.:

↑: $A + B = -F \sin\alpha$

→: $\underline{\underline{C = F \cdot \cos\alpha}}$

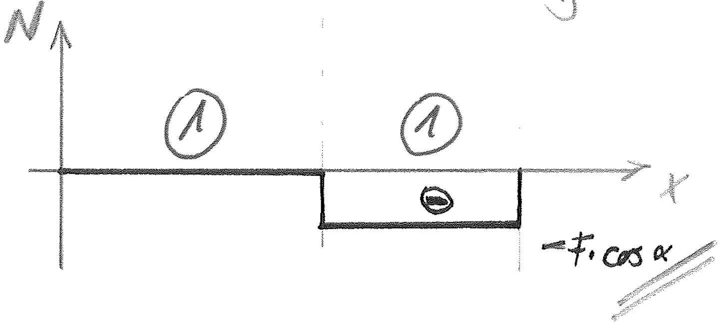


pos. Schnittufer: $\underline{\underline{N \equiv 0}}$



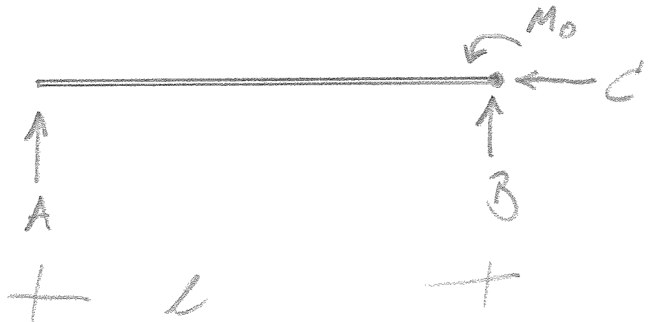
neg. Schnittufer:

$\underline{\underline{N}} = -C = \underline{\underline{-F \cdot \cos\alpha}}$



---

ii)



GG-Bed.:

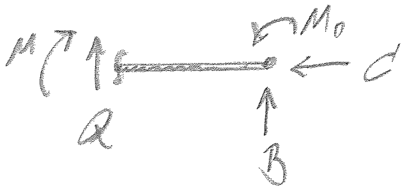
↻: $M_0 = A \cdot l \rightarrow \underline{\underline{A = \frac{M_0}{l}}}$

↑: $A + B = 0 \rightarrow \underline{\underline{B = -\frac{M_0}{l}}}$

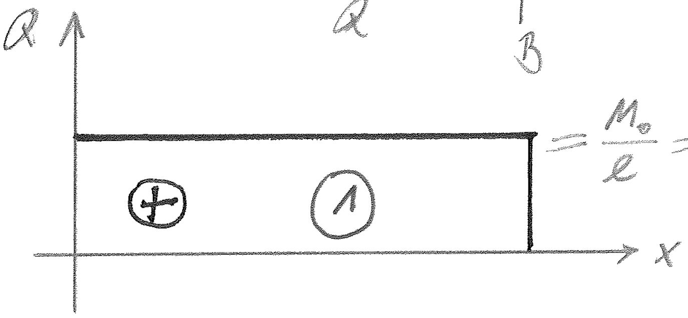
1

$\Rightarrow \underline{\underline{Q = A = \frac{M_0}{l}}}$

1

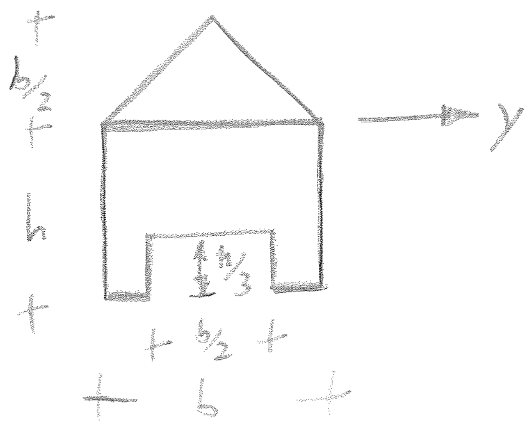


$\Rightarrow \underline{\underline{Q = -B = \frac{M_0}{l}}}$



$= \frac{M_0}{l} = \underline{\underline{\text{konst.}}}$

3.



$$I_y^{ges} = I_y^{\Delta} + \left(\frac{1}{3}\,\frac{b}{2}\right)^2 \cdot \overbrace{\frac{1}{2}\,\frac{b}{2}\,b}^{A^{\Delta}}$$ (1) (1)

$$+ I_y^{\square} + \left(\frac{1}{2}h\right)^2 \cdot bh$$ (1) (1)

$$- I_y^{*} - \left(\frac{2}{3}h + \frac{1}{2}\frac{h}{3}\right)^2 \cdot \frac{b}{2}\cdot\frac{h}{3}$$ (1) (1) (1)

$$\Rightarrow I_y^{ges} = \frac{1}{36}b\left(\frac{b}{2}\right)^3 + \frac{b^2}{36}\cdot\frac{1}{4}b^2 + \frac{1}{12}bh^3 + \frac{1}{4}bh^3 +$$

$$- \frac{1}{12}\,\frac{b}{2}\left(\frac{h}{3}\right)^3 - \left(\frac{5}{6}h\right)^2\cdot\frac{1}{6}bh$$

$$= \frac{1}{288}b^4 + \frac{1}{144}b^4 + \frac{1}{3}bh^3 - \frac{1}{648}bh^3 - \frac{25}{216}bh^3$$

$$= \frac{3}{288}b^4 + \left(\frac{216}{648} - \frac{1}{648} - \frac{75}{648}\right)bh^3$$

$$= \frac{1}{96}b^4 + \frac{35}{162}bh^3$$ (2)

4.

$$\frac{S_2}{S_1} = e^{\mu\alpha} = e^{\frac{1}{\pi}\pi} = e$$ (1) (1)

5. Fahrzeugbewegung in 3 Zeitabschnitte:

1.: $t = 0 \ldots 100\,s$
2.: $t = 100 \ldots 300\,s$
3.: $t = 300 \ldots 400\,s$

• „Beschleunigung" ist jeweils die Steigung im v-t-Diagramm.

$$> a_1 = \frac{6}{100}\,\frac{m}{s^2} \quad (1) \qquad > a_2 = 0 \quad (1) \qquad > a_3 = \frac{d}{dt} v_3$$

Detail 3. Abschnitt:

$$v_3 = k(t - 400\,s)^2 \qquad \text{„quadrat. Parabel"} \rightarrow k = ?$$

aus Diagr.: $v_3(300\,s) \overset{!}{=} 6\,\frac{m}{s} = k(-100\,s)^2$

$$\Rightarrow k = \frac{6}{10000}\,\frac{m}{s^3} \Rightarrow v_3 = \frac{6}{10000}\,\frac{m}{s^3}(t - 400\,s)^2 \quad (2)$$

$$\rightsquigarrow a_3 = \frac{dv_3}{dt} = \frac{6}{5000}\,\frac{m}{s^3}(t - 400\,s) \quad (1)$$

• „Weg" ist Fläche unter der v-t-Kurve im jew. Abschnitt.

$$> s_1 = \int v_1(t)\,dt = \frac{1}{2}\,100\,s \cdot 6\,\frac{m}{s} = 300\,m \quad (1)$$

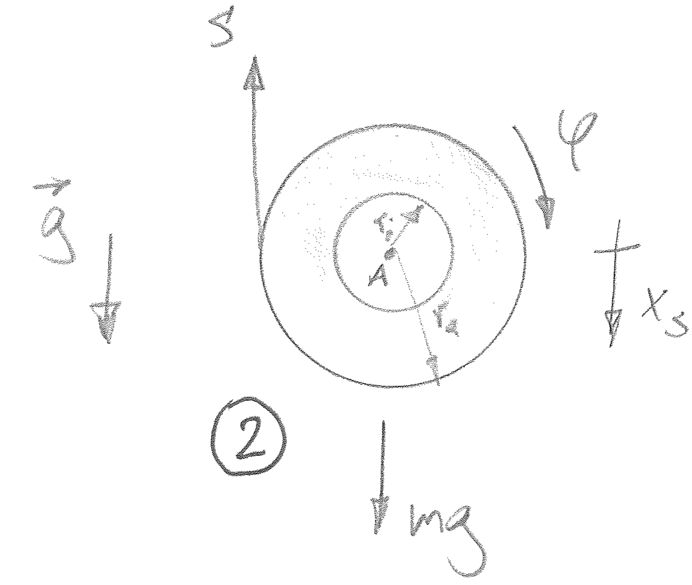
$$> s_2 = 6\,\frac{m}{s} \cdot 200\,s = 1200\,m \quad (1)$$

$$> s_3 = \int_{300\,s}^{360\,s\,!} v_3(t)\,dt = \frac{6}{10000}\,\frac{m}{s^3}\int_{300\,s}^{360\,s}(t - 400\,s)^2\,dt \quad (2)$$

$$= \frac{2}{10000}\,\frac{m}{s^3}\left[(t - 400\,s)^3\right]_{300\,s}^{360\,s} = \frac{2}{10000}\,\frac{m}{s^3}\left(-64000\,s^3 + 1000000\,s^3\right)$$

$$(1) = 187{,}2\,m \qquad \Rightarrow s^*_{ges} = s^*_{6\,min} = s_1 + s_2 + s_3 = 1687{,}2\,m$$

6. Dynamik



- $\Theta_A = \frac{1}{2} m (r_a^2 + r_i^2)$ (1)
- $\varphi \cdot r_a = x_S$

$\ddot{\varphi} \cdot r_a = \ddot{x}_S$ (1)

i) bsp.weise mithilfe des Energiesatzes:

(1) $E_{pot} = m g x_S$

(1)+(1) $E_{kin} = \frac{1}{2} m (\dot{x}_S)^2 + \frac{1}{2} \Theta_A \left(\frac{\dot{x}_S}{r_a}\right)^2$; $\dot{x}_S$: Geschw. des Schwerpkt.

$$2 \cancel{m} g x_S \overset{!}{=} m (\dot{x}_S)^2 + \frac{\Theta_A}{r_a^2} (\dot{x}_S)^2$$

$$= \cancel{m} (\dot{x}_S)^2 + \frac{\cancel{m}}{2} \frac{r_a^2 + r_i^2}{r_a^2} (\dot{x}_S)^2 = \left[\frac{3 r_a^2 + r_i^2}{2 r_a^2}\right] (\dot{x}_S)^2$$

$$\Rightarrow \frac{2 r_a^2}{3 r_a^2 + r_i^2} \cdot g \cdot x_S = \dot{x}_S^2$$ (1)

ii) Bewegungsgesetze: $\downarrow: m \ddot{x}_S = mg - S$ (1) ; $\curvearrowright: \Theta_A \ddot{\varphi} = S r_a$ (1)

einsetzen: $\frac{m (r_a^2 + r_i^2)}{2 r_a} \ddot{x}_S = (mg - m \ddot{x}_S) r_a$ (1)

(1) $\rightarrow \ddot{x}_S = \frac{2 r_a^2}{3 r_a^2 + r_i^2} g$ > analog auch aus i) durch Diff. !