Musterlösung TM-Klausur – 18.04.2015

1. Flächenmoment

$$I_y = I_y^{\square} + \left(\frac{h}{2}\right)^2 bh - \left[I_y^{\triangledown} + \left(\frac{1}{3}\frac{b}{2}\right)^2 \cdot \frac{1}{2} b \frac{b}{2}\right]$$

$$= \underset{(1)}{\frac{bh^3}{12}} + \underset{(2)}{\frac{bh^3}{4}} - \underset{(1)}{\frac{b\left(\frac{b}{2}\right)^3}{36}} - \underset{(3)}{\frac{b^4}{144}}$$

$$= \frac{bh^3 + 3bh^3}{12} - \underset{(2)}{\frac{b^4 + 2b^4}{288}} = \underset{(1)}{\frac{bh^3}{3} - \frac{b^4}{96}}$$

Σ 10

2. Spg-tensor

$$\underset{\sim}{\sigma} = \begin{bmatrix} -1 & 4 \\ 4 & 5 \end{bmatrix} \text{MPa}$$

a) $\sigma_1 = 7\,\text{MPa}$ (2)

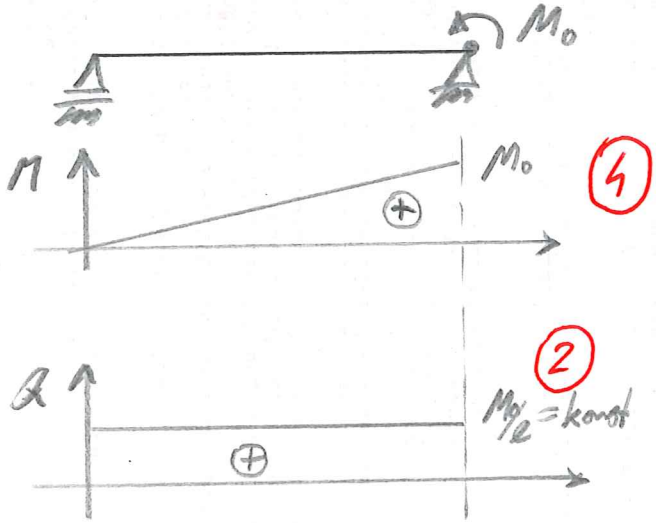
$\sigma_2 = -3\,\text{MPa}$ (2)

b) $\varphi^* = \frac{1}{2}\arctan(\cdot) = \underset{(1)}{-26{,}57°}$

c) $\underset{\sim}{\sigma}^* = \underset{(2)}{\begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix}^T} \cdot \underset{\sim}{\sigma} \cdot \underset{(2)}{\begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix}} = \underset{(1)}{\begin{bmatrix} 3{,}9641 & 4{,}5981 \\ 4{,}5981 & 0{,}0359 \end{bmatrix}} \text{MPa}$

Σ 10

3. Schnittgrößen

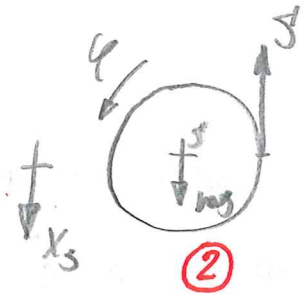


(4)

(2)

Σ 6

4. Starkörperdynamik:



$x_S = r\varphi$

$\ddot{x}_S = r\ddot{\varphi}$ (1)

$\Theta_S = \frac{1}{2} m r^2$ (1)

a) Energiesatz:

$-E_{pot} = mgx_S$ (1)

$E_{kin} = \frac{1}{2} m \dot{x}_S^2 + \frac{1}{2} \Theta_S \left(\frac{\dot{x}_S}{r}\right)^2$ (2) $\rightsquigarrow E_{pot} + E_{kin} = 0$ (2)

$2mgx_S = m\dot{x}_S^2 + \frac{1}{2} m r^2 \frac{\dot{x}_S^2}{r^2} \Rightarrow \dot{x}_S^2 = \frac{4}{3} g x_S$ (1)

b) Differentiation: $\frac{d}{dt}\left\{\dot{x}_S^2 - \frac{4}{3} g x_S = 0\right\}$

$2\dot{x}_S \ddot{x}_S - \frac{4}{3} g \dot{x}_S = 0 \rightarrow \ddot{x}_S = \frac{2}{3} g$ (2)

Σ 12

5. Biegelinie

$q(x) = q_0 + ax + bx^2$ mit (1) $q' = a + 2bx$ und $q'(0) = 0$

$q(\ell) = 3q_0 = q_0 + a\ell + b\ell^2$

$\Rightarrow a = 0$; $3q_0 = q_0 + b\ell^2 \Rightarrow b = \frac{2q_0}{\ell^2}$

$\rightsquigarrow q(x) = q_0 + \frac{2q_0}{\ell^2} \cdot x^2$ (1)

„x läuft von links nach rechts."

$\rightarrow EIw'''(x) = q(x) = q_0 + \frac{2q_0}{\ell^2} \cdot x^2$ (1)

$EIw''(x) = -Q = q_0 x + \frac{2q_0}{3\ell^2} x^3 + C_1$ (1)

$$EI\,w''(x) = -M = \frac{1}{2}q_0 x^2 + \frac{q_0}{6\ell^2}x^4 + C_1 x + C_2 \quad (1)$$

$$EI\,w'(x) = \frac{1}{6}q_0 x^3 + \frac{q_0}{30\ell^2}x^5 + \frac{1}{2}C_1 x^2 + C_2 x + C_3 \quad (1)$$

$$EI\,w(x) = \frac{1}{24}q_0 x^4 + \frac{q_0}{180\ell^2}x^6 + \frac{1}{6}C_1 x^3 + \frac{1}{2}C_2 x^2 + C_3 x + C_4 \quad (1)$$

Randbed.: i) $w(x=0) = 0$

ii) $w'(x=0) = 0$ (2)

iii) $w'''(x=\ell) = 0$

iv) $w''(x=\ell) = 0$

Einsetzen: → i) $C_4 = 0$

→ ii) $C_3 = 0$ (2)

→ iii) $q_0\ell + \frac{2q_0}{3\ell^2}\ell^3 + C_1 = 0 \rightarrow C_1 = -\frac{5}{3}q_0\ell$

→ iv) $\frac{1}{2}q_0\ell^2 + \frac{1}{6}q_0\ell^2 - \frac{5}{3}q_0\ell^2 + C_2 = 0 \rightarrow C_2 = q_0\ell^2$

$$\Rightarrow w(x) = \frac{1}{EI}\left\{\frac{1}{24}q_0 x^4 + \frac{q_0}{180\ell^2}x^6 - \frac{5}{18}q_0\ell x^3 + \frac{1}{2}q_0\ell^2 x^2\right\} \quad (1)$$

Σ 12